Mathematical Knowledge for Teaching: FoM and IME Conference
Notes of Summary by Wayne Harvey

I want to Poke you a bit.

During this session, ask yourself, "What might I do in addition to 'business as usual'."

It's been a very engaging 2+ days. I suspect we're all a bit worn out.

I missed a lot with what I heard, looked for structure, experimented. I did find patterns. But precision….

Two things to do:
1. I don't want to do a summary - - no point to that. But I want to try to hold up a kind of mirror to reflect back few places of energy.
2. Ask "what next?" or "what else?"
   The value of the interchanges made it plenty successful for me, so these questions will be just to experiment.

Okay, about that mirror. If I distort your view, quote you out of context, change your words or intended meaning . . .

First I'll organize our conversations into the following four categories:
1. Language we use
2. Positions/Desired/Emotions
3. Institutions/Beliefs/Knowledge (This morning put a little light on these labels.)
4. Questions/Recommendations

Then we can have a discussion in which we might reflect on:
   What are important goals?
   What is important work to do?
   How can the work be sustained or institutionalized?

1. Language

We kept missing, then questioning, some of the language we use to identify the "stuff" that matters in our work. Precision?
Just this morning we heard more of this questioning our language:

Emina: "Belief" vs. "Knowledge" Mike also said this.

Dick laid the first marker down for this bit of reflection.
Things I heard that were important:
   Deep understanding Rich understanding
   Profound understanding
   Solid understanding

From Dan and Pat: Collective Knowledge
   Individual Knowledge

Organizational Knowledge or Institutional Knowledge

From Pat Wilson and Kathy:
Mathematical proficiency
Mathematical activity
Mathematical work of teaching

From Marilyn: Mathematical integrity, intimacy

Then there's meaningful mathematics, essential mathematics. As Wade pointed out, there is even applied mathematics, and statistics.

There is fundamental importance.
Lots of important stuff, neat stuff, and tough stuff as Gail called it.
And Marilyn, and others, said don't forget to mess around.
Unless, as Tom said, it's BSTC (beyond the scope of this course).
Glenn said, as did 10-20 others, 'think like a mathematician.'
And that word "mathematician" got us in a bit of trouble. Glenn clarified.
And experience before formality. Glenn again. And Al and Ryota and others.
Think deeply (or solidly) of simple things. Thank you Arnold Ross.

So MKT? Emina says, "I have no idea what this is."
Thanks to Rebecca I have the Leanhardt quote: "it is more important that you do it the same way than what you think is the right way."
Of course, there's the question of whose "same way."

Maybe trying to create a taxonom of meanings and highlighting relationships would be a productive path.

2. Positions/Desires/Emotions

(Passion is a good thing. It's worth asking how to utilize it.)

Sarah: We're not trying to prove anything. We already know everything we love about Focus on Mathematics is true.

Glenn: Mathematics is the art of figuring things out. "These are not all consistent. That's beside the point. All of this is true."

Dick: Surprises in mathematics are useful. Miracles matter.

(Then there was something about $e^{\pi\iota} = -1$ and god. Thank you, Eileen.)

Speaking of god . . .
John noted this is a very top down approach. There are out there a lot of really good teachers.

Mike suggested we need an evidence base of why preparing teachers matters.

Herb: Only the teachers themselves can raise their professional status.

Dick says "I would like teachers to know . . . "

And along side that is Sarah's comment "Allow teachers to feel safe at not knowing."
No inconsistency here.

Dick stated we will need massive PD relative to the Common Core.

Bill suggests people need to see what's unifying at the secondary level.
Unpack for elementary. Pack it up for secondary.

Tom points out "Beauty, interest, taste can be given short shrift."

Yvonne captured a perspective very nicely: "That's not where I imagined I was going, but that's where I wished I had imagined I was going."
And relevant to that is Ginny's comment: "I'm liking the badge idea more and more."

OK one more here:
Kihui observes "in this country, different people will have different opinions" (on anything, on everything).

I was thinking it might help in the field to work harder at distinguishing opinions, wishes, beliefs, evidence, findings, conjectures, theorems, proofs.
We need to distinguish and aggregate our knowledge better.

3. Institutions/Beliefs/Knowledge (I'll just touch the waters here)

Pat Wilson notes teaching to learn mathematics is different from teaching mathematics.

John suggests: Give teachers sufficient mathematics and the right environment and time so that they can spend their career learning good mathematics. (This was also central to points Herb mentioned.)

From Ryota: Help teachers "struggle productively."
Al talked about this as not being about where we get to, it's how we get there. (That's when he missed his exit.)

From John again:
Find really great teachers and link them together in a local network. (Bill has started a forum focusing on Common Core tasks.)

Juliana asks: "and what about the not-so-great teachers?"

I want to put two other comments side-by-side because they present an interesting yin-yang.

Dan Think of systems in which instruction must sit. Focusing on a new piece of really nice mathematics may be a distraction.

Pat Mathematical experiences and student performances don't have to be bound together. There's the idea of keeping mathematics alive.

Lastly here, a few comments on the Common Core:

- Bill noted the importance of focus
- Herb noted that this is a huge political opportunity
- And the group agreed that implementation is of critical importance.

Joan suggested we could learn a lot through a case-study over time of a successful state.

And this brings to mind Mary's observation from a large urban district: There can be a lot of change with very little results.

5. **Questions/Recommendations** (Just examples, but quite a bit of coherence here.)

(Sol) How do teachers understandings of what's important in mathematics as a discipline change as they experience these programs?
(Dan) And impact on practice?

(Bill) How are the CCSS mathematical practices related to or components of mathematical knowledge for teaching? (map the landscape)

(Pat W) What is the nature of mathematics you need to help others learn mathematics?
(Tom) How does time available, importance, difficulty impact the answer?
(Wade) How does mathematics software impact the answer?

(Dan, Pat) How can we create frameworks for organizational knowledge or group knowledge that go beyond knowledge as only residing in an individual's head?
(And can we apply that thinking to our own field and efforts such as this meeting?)

(Tom) And every discipline, especially mathematics, has structure and frameworks. Tom asks how does greater engagement with these lead to learning outcomes of a different sort.

(Ryota) What does it mean to do well in a program designed to have teachers experience mathematics as mathematicians do? (And impact on classroom practice?)
(John) How much variation is there in the way mathematicians think mathematically? And do we want that necessarily to be our goal for students? (Glenn elaborated here.)

(Gail) What factors influence success (after we've defined what that is) in programs like PCMI, PROMYS, FoM? Again, what are effects of practice? How do we collect evidence (or data)? (Mike: what counts is evidence?)

(John) And if teaching is at least somewhat an art, how do you measure that?

(Todd) To what extent are outcomes dependent on individual dispositions?

(Alejandro) How can work of this kind be replicated?

(Joan) What makes for good implementation of such programs and how can we increase capacity to do them?

(Dan & Juliana)

How can we impact systems? How can we galvanize larger numbers in the field?